Int. J. Heat Mass Transfer. Vol. 8, pp. 921–935. Pergamon Press 1965. Printed in Great Britain

LAMINAR FLAME PROPAGATION IN DROPLET SUSPENSION OF LIQUID FUEL

YUKIO MIZUTANI and MITSUNOBU OGASAWARA
Department of Mechanical Engineering, Osaka University, Miyakojima-ku, Osaka, Japan

(*Received* 15 *December* 1964)

Abstract—As a first step in the clarification of the mechanism of flame propagation in the spray of a liquid fuel, a study was conducted into the case of laminar flame propagating through the droplet suspensions of a liquid fuel, having uniform diameters. Initially, in order to examine the nature of flame propagation, tetralin droplet suspensions having approximately uniform diameters were prepared, and their concentration at the lower limit of flammability was examined, and, at the same time, the appearance of the flame and the process of its propagation were observed. On the basis of this observation, a model of flame propagation was proposed, in view of the phenomenon that the droplets burning at the flame front heat the unburned mixture, then the unburned droplets are ignited after the lapse of a certain ignition lag. Using this model, a theoretical analysis was carried out, and the velocity of flame propagation and the concentration at the lower limit were calculated. The predicted values showed satisfactory agreement with experimental results, at least qualitatively. From this analysis, it was found that sedimentation of droplets exerts great influence on the process of flame propagation, and that, in case the mixture containing droplets is diluted by nitrogen, its effect on the lower limit depends greatly upon the droplet diameter, as had already been found experimentally.

NOMENCLATURE

A^*,	constant;
a,	thermal diffusivity, $\lambda/\gamma_g c_v$;
b,	distance between A plane and the flame front;
b^*,	constant;
C,	concentration;
c,	specific heat;
D_f,	diameter of the diffusion flame;
d,	droplet diameter;
E,	activation energy;
H,	heat of combustion of fuel;
$K = 1{\cdot}0 + 0{\cdot}30\, Pr^{1/3}\, Re^{1/2}$;	
K_b,	burning constant of droplet;
k^*,	constant;
$k_1 \sim k_4$,	constants;
L,	distance between droplets in the unburned mixture;
Nu,	Nusselt number;
Pr,	Prandtl number;
$\mathscr{R}$,	universal gas constant;
Re,	Reynolds number;
r,	heat of vaporization of fuel;
T,	absolute temperature,
t,	time;
V_f,	propagation velocity of the flame;
V_t,	terminal settling velocity of droplet;
w_A,	velocity of A droplet;
x,	distance;
Z,	function related to chemical ignition lag;
Z^*,	constant;
z,	stoichiometric mass ratio of oxygen to fuel.

Greek symbols

α_c,	coefficient of heat-transfer;
γ,	specific weight;
η,	dimensionless time, t/τ;
θ,	constant (<1);
μ,	coefficient of viscosity;
$\xi = x/\sqrt{t}$;	
τ,	burning lifetime, d_0^2/K_b.

Subscripts

0,	initial condition;
A,	A droplet (A plane);
B,	B plane;
d,	droplet (B droplet);

eq,	equilibrium condition;
f,	flame front;
g,	gas (air);
ig,	ignition;
l,	liquid fuel;
min,	lower limit of flammability;
O,	oxygen;
p,	at constant pressure;
v,	at constant volume.

INTRODUCTION

It is needless to say that knowledge about flame propagation in liquid fuel sprays is of vital importance, especially in relation to the stability of flames in such apparatuses as gas turbine combustors, where liquid fuels are burned under a high load, and in the burners of heavy oil-burning boilers. As already found out experimentally, however, the nature of flame propagation in sprays greatly depends on the microscopic structure of flame front and the minute phenomena occurring in flame front [1], and, accordingly, it is made exceedingly complex, and, besides, in certain cases, it presents most varying aspects. For these reasons, very few theoretical investigations have so far been conducted in this field. Much less have been those theoretical analyses considering the microscopic structure of flame.

As a first step in the study for the clarification of the phenomenon of flame propagation in sprays, the authors took up the simplest and most ideal case—the case when a laminar flame propagates one-dimensionally through a droplet suspension of liquid fuel of a pure substance having a uniform diameter—and examined the mechanism of propagation, then carried out a theoretical analysis to predict the flame propagation velocity and the concentration at the lower limit of flammability.

EXPERIMENT

Tetralin droplet suspension having an approximately uniform diameter was prepared in order to examine the nature of flame propagation, and its lower limit concentration of flammability was measured, and, at the same time, the appearance of flame and the process of its propagation were observed.

Apparatus and method

Tetralin droplet suspension having a uniform diameter was prepared by the method in which tetralin spray atomized by swirl atomizers was assorted in an elutriation wind tunnel, and such droplets only within a limited range of diameter were made to fall selectively into a glass-made vertical combustion tube. A sketch of the experimental apparatus is shown in Fig. 1. In the apparatus, the fuel is injected into the container 2 from one pair or two pairs of swirl atomizers 1, opposing each other. Part of the spray, of which velocity is reduced as a result of collision falls in the elutriation wind tunnel 3, and, by the difference in the velocities of sedimentation, the droplets are assorted, only those droplets having a certain limited range of diameter being taken into the combustion tube 9. Into the upper entrance of the glass-made combustion tube (55 mm in inner diameter, and 1200 mm in length) provided in the lowermost part, is inserted the throttle 8, 40 mm in inner diameter, by which the sectional area wherein droplets falls is defined, and, at the same time, the droplets are prevented from being stuck on the wall surface. In order to prevent

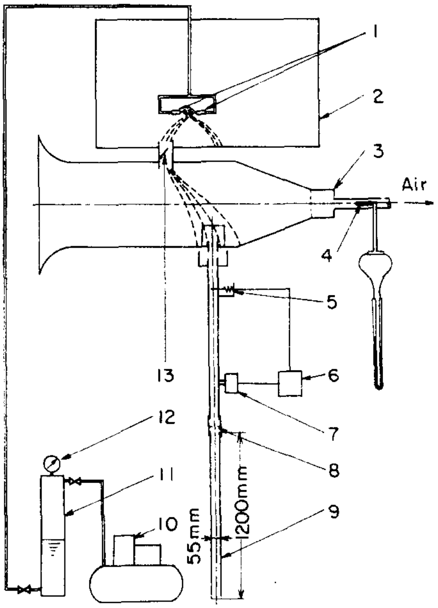


Fig. 1. Experimental apparatus.

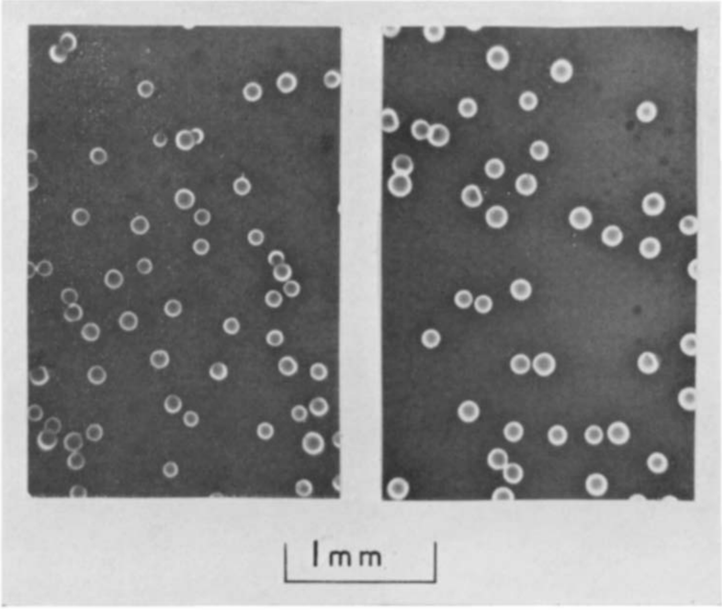


FIG. 2. Photographs of droplets.

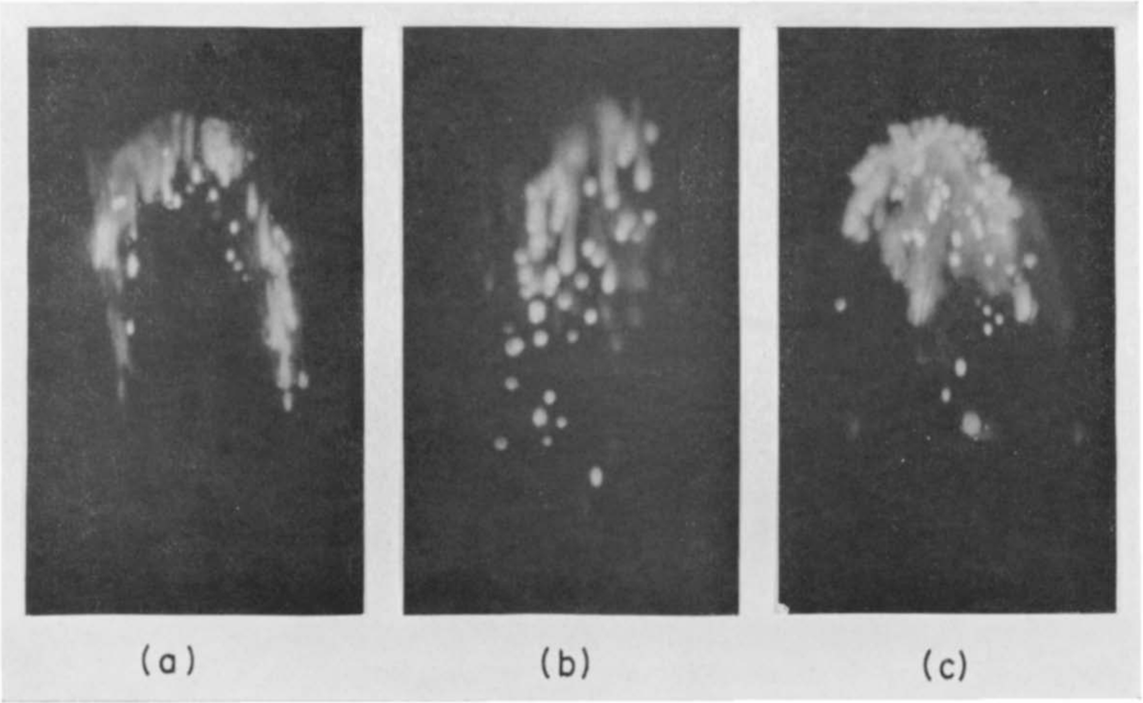


FIG. 4. Photographs of flames.

Fuel: tetralin. Time of exposure: 1/250 s. Mean diameter of droplets: (a) 140 μ, (b) 170 μ, (c) 100 μ. Concentration: (a), (b) near the lower limit, (c) a little thicker. Upper end of the tube: (a), (c) open, (b) closed.

the invasion of flame into the wind tunnel, a magnet shutter 5, driven by a photo-electric device 7 and relay 6, is equipped in the upper part of the combustion tube. This shutter is so constructed that it may also be set in motion by hand, being used both for examining the effect of the condition of the upper end of the combustion tube on the process of flame propagation and for measuring the concentration of the mixture. The concentration of droplet suspension inside the combustion tube may be adjusted by the throttle valve 13 and fuel injection pressure, and, for this purpose, the fuel-pressurizing tank 11, air-compressor 10 and pressure gauge 12 are installed. 4 is the pitot tube for measuring the velocity of air flow inside the wind tunnel.

For the lower limit of flammability, the lowest concentration that the flame propagates from the bottom to the top of a combustion tube, 55 mm in inner diameter and 1200 mm in length, was adopted. For ignition, a small burner was used. For measuring the droplet diameter, the method using magnesium oxide-coated slides was adopted [2]. Some examples of the photographs of the droplets is given in Fig. 2. The weight of droplets contained in a unit volume of the mixture (concentration of the mixture) was measured by sampling and weighing the droplets in a definite volume. The appearance of flame and the process of its propagation were observed by direct observation and photographing (direct photograph).

The fuels used in the experiment were tetralin and No. 2 light oil. By this apparatus, a mixture of very much higher concentration above the lower limit of flammability could not be obtained, the range of droplet diameter being 100 ~ 200 μ.

Concentration at lower limit of flammability

The concentration at the lower limit when the flame propagates upwards was measured pertaining to the cases of the upper end of the combustion tube (magnet shutter 5 shown in Fig. 1) being opened and being closed, by using two kinds of fuel, tetralin and light oil. The result obtained in these observations is as shown in Fig. 3. Burgoyne and Cohen [3] have conducted measurement of the concentration at the lower limit under a similar condition by preparing small droplet suspensions having uniform droplet diameters (7 ~ 55 μ) through condensation of tetralin vapor on nuclei. For the convenience of comparison, the result obtained by them was also given in the same diagram.

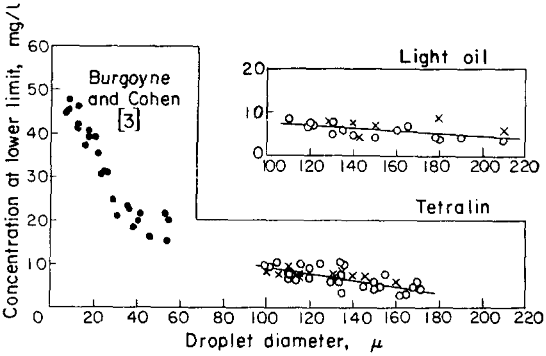


FIG. 3. Lower limit of flammability. ○ Upper end of the tube is opened, × closed.

From this diagram, it is known that the measured values obtained in this experiment are closely related to Burgoyne and Cohen's result and that the concentration at the lower limit would decrease rapidly in proportion as the droplet diameter increases (up to 100 μ), subsequently continuing to decrease almost linearly. Practically no difference is seen in the concentration at the lower limit between the cases the upper end of the combustion tube is opened and closed. The result obtained with tetralin, when compared with that obtained with light oil, did not likewise indicate any remarkable difference, in so far as the present experiment was concerned.

Observation of flame

Figure 4 shows direct photographs of flames propagating upwards inside the combustion tube. It is seen that each droplet burns, surrounded by an independent flame and that no combustion takes place in the interdroplet space. Observed macroscopically, the flame appears to be composed of a cluster of yellow or yellowish-red points, independent and brightly shining. From this, it is assumed that each droplet independently carries out heterogeneous combustion and that the flame propagates discontinuously from one droplet to another. Besides, for the relatively short time of exposure, i.e. 1/250s, the traces of flight of droplets near the flame front are seen to be rather long, which suggests that the droplets, immediately after ignition, are accelerated due to thermal expansion of the air. It is also shown that, while the flame at the instant of droplets being ignited is weak and dark, it gradually grows into a stronger and brighter flame—unsteady-state combustion of droplets.

From the absence of any essential difference between these three photographs, it is understood that opening or closing of the upper end of the combustion tube, and slight increase in the concentration of the mixture do not exert any major influences on the propagation mechanism of flame. Moreover, the fact that the flame assumes a shape similar to an inclined plane, though it is not clear in these photographs, constitutes a characteristic feature distinguishing it from combustion of gaseous fuels.

PROPAGATION MECHANISM

Observations on propagation mechanism

In spray combustion, it is generally understood that flame propagates in two different mechanisms, relay transfer and homogeneous transfer, and the mechanism intermediate between them [1]. The former, as examined in the aforementioned experiment, is a mechanism in which flame propagates discontinuously from one droplet to another, while the latter is one in which flame propagates continuously through the interdroplet space filled with mixture of fuel vapor and air. As described above, Burgoyne and Cohen [3] observed the construction of flame front and its mechanism of propagation by preparing small tetralin droplet suspensions having approximately uniform diameters. Their observation, if interpreted from the viewpoint of the propagation mechanism of flame, will indicate that, in case the diameter is above 40 μ, relay transfer of flame will take place, but, over the range of 40 $\sim$ 20 μ, the mechanism of propagation changes, homogeneous transfer being seen below 20 μ. Below 10 μ, the droplets will completely vaporize before combustion, the appearance being that of gaseous premixed flame.

On the other hand, as realized from Fig. 3, the concentration at the lower limit of flammability is rapidly reduced with increase of droplet diameter. Also, according to Burgoyne and Cohen [3], the velocity of flame propagation is greater, nearly twice as big, while less effect is given by dilution of the inert gas around 40 μ where relay transfer takes place, than in case of diameter being below 10 μ where the burning characteristics approach those of homogeneous gaseous mixture. The fact that flame propagation is more facilitated in proportion as the droplet diameter increases may not be explained by other than a change in the propagation mechanism of flame.

From this point of view, a hypothesis on the propagation of flame will be set up, with emphasis placed on the salient features of relay transfer, and, on the basis of that hypothesis, a theoretical analysis will be undertaken below. As an approach to the analysis, an attempt will be made in the following paragraph to make a model of propagation mechanism of flame.

Model of propagation mechanism

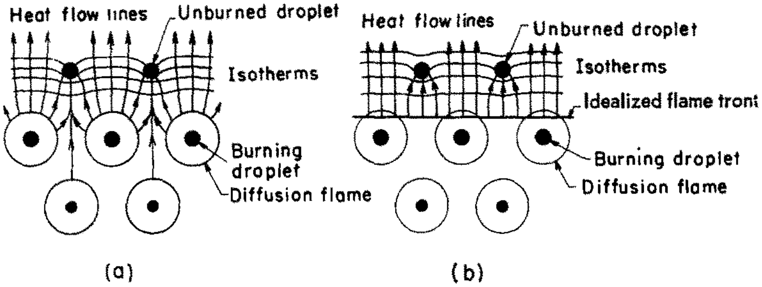
Three main procedures in making a model of the propagation mechanism in such a way that a theoretical analysis may be facilitated are as follows:

(1) To find out main factors governing the process of flame propagation;

(2) To approximate the positions of droplets irregularly scattered in the air to an appropriate orderly arrangement;

(3) To introduce an appropriate hypothesis to treat the process of flame propagation one-dimensionally.

From the features of relay transfer as described in the above, it may be assumed that flame propagates through a repetition of a process as follows. Namely, as droplets are ignited, exceedingly high-temperature flames are formed around them, and, through heat transfer therefrom, the air in the unburned area is heated, and, thus, unburned droplets are ignited after a certain ignition lag. Accordingly, it may be assumed that heat transfer from the flames



(a) (b)

FIG. 5. Thermal field near the flame front.

surrounding droplets and ignition lag of unburned droplets consitute the main factors governing the process of flame propagation. Besides, consideration will perhaps have to be given to the sedimentation of droplets due to the gravity force and the effects of thermal expansion of gas.

In regard to the arrangement of droplets mentioned in (2) above, two kinds of arrangement, simple cubic lattice and body-centered cubic lattice, may be assumed. For simplicity, the former will be adopted here.

Now, the phenomenon will be one-dimensionalized to make its theoretical handling easier. Here to be considered are heat transfer from flame and a resulting thermal expansion of gas. Supposing that droplets are arranged orderly, the phenomenon of heat transfer from the flames surrounding burning droplets to the unburned area may be illustrated schematically as Fig. 5(a). It is seen that the directions of heat currents are approximately parallel at a little distance from the flames, and that, consequently, the isothermal surface approximates a plane. Accordingly, it is assumed, as shown in Fig. 5(b), that no major difference will be caused in the thermal field of the positions of the unburned droplets next to the burning droplets even if the flame around each droplet were replaced by a continuous plane-shaped flame. For simplicity, this flame front is assumed to be provided only with the action of a constant-temperature heat source and it is also assumed that the burnt area is filled with the air having the same temperature with the flame front.

Thus, heat transfer from the flame front to the unburned area may be calculated, and, accordingly, the ignition time of unburned droplets is estimated by an appropriate method. Now, if it is assumed that the plane-shaped flame jumps by a distance equal to the spacing between droplets at the instant of ignition, the velocity of propagation may be calculated from the time of ignition. If the assumption is made that, in order for a stable propagation to be carried out, ignition of unburned droplets has to be carried out considerably before burning droplets are burnt out, the lower limit of flammability may be defined. Since, besides, sedimentation of droplets constitutes a considerably important factor, it is necessary to designate the direction of flame propagation in analysis.

THEORETICAL ANALYSIS

It is extremely difficult to calculate the temperature rise of unburned mixture due to heat conduction from the flame front, and the movement of the mixture due to thermal expansion by using a set of strict equations of motion of the entire system, including droplets and gas, and, therefore, calculation will be undertaken regarding two ideal cases; when thermal expansion of gas is ignored, and when thermal expansion of gas under a constant pressure is assumed. The former stands on the assumption that the inertia of gas has an exceedingly great effect, while the latter ignores it and the drag force exerted on gas by droplets. Also, in respect to the direction of flame propagation, upward propagation receiving the greatest effect from gravity force and horizontal propagation under the least effect from it will be treated.

Here, the case of upward one-dimensional propagation of flame will be treated as a typical case, with thermal expansion entirely ignored. For simplicity, other factors, considered not so essential, will be ignored here.

Model of flame propagation

As shown in Fig. 6, the case is assumed here when flame propagates one-dimensionally in a direction normal to the lattice surface, through the droplet suspension of liquid fuel having a uniform diameter d_0, distributed on simple cubic lattice arrangement in still air (lattice distance L). If it is assumed that the phenomenon is perfectly one-dimensional, droplets within the lattice surface A (hereinafter called A droplets) will be simultaneously ignited at a certain instant (this is taken as the origin of time t, and is put as $t = 0$), flame is formed around each droplet. In order to make possible a one-dimensional treatment, this flame surrounding each droplet is replaced by a hypothetical plane-shaped flame front, spaced by b from A plane. The plane next to A plane, where unburned droplets exist, are named B plane. As the flame front appears, the temperature of the unburned air will rise due to heat conduction from it. However, it is assumed that pressure rise only is brought about and no flow of gas due to thermal expansion is caused. Here, the following assumptions are made:

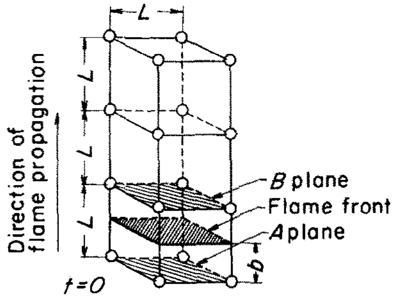


FIG. 6. Arrangement of droplets.

(1) Flame has only the action as a heat source of a constant temperature. The flame temperature T_f and the distance from A plane to the flame front b is kept at a fixed value during the burning lifetime of A droplets.

(2) The liquid fuel is a pure substance.

(3) The temperature of the mixture T_0 is sufficiently lower than the boiling point of the fuel, and, consequently, existence of fuel vapor in the unburned mixture may be ignored.

(4) The instant when fuel droplet (A droplet) is ignited is put as $t = 0$, and the diameter at this time is taken as d_0. Then, the subsequent droplet diameter d_A will change according to the relation of $d_A^2 = d_0^2 - K_b t$. Where K_b is burning constant.

(5) Each of the droplets, until the instant of ignition, maintains the initial velocity of sedimentation (the terminal settling velocity V_t) and during the burning lifetime $\tau(= d_0^2/K_b)$, the velocity decreases linearly (Fig. 7).†

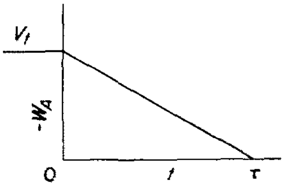


FIG. 7. Velocity of A droplet.

(6) The effect of pressure rise due to temperature rise, exerted on the equilibrium temperature (wet-bulb temperature) of droplets T_{eq} and chemical ignition lag is ignored.

(7) The specific weight of the liquid fuel is assumed to be kept constant and any decrease in droplet diameter due to evaporation until the instant of ignition is ignored.‡

(8) Radiative heat transfer from flame to unburned droplets is ignored.‡

Now, as shown in Fig. 8, on the direction of propagation, the axis of coordinates x is taken, and its origin O is so fixed that it may be located on the position of the flame front at the instant

---

† This assumption does not seem very unreasonable, in view of the fact that droplet diameter has only slightly changed until they are ignited and that, after ignition, the square of diameter decreases with time linearly and, moreover, that the velocity of sedimentation of droplets is approximately in proportion to the square of droplet diameter.

‡ That the evaporation of droplets until ignition and the effect of radiative heat transfer from flame to unburned droplets are so small that may be ignored, is certified by a more detailed theoretical analysis.

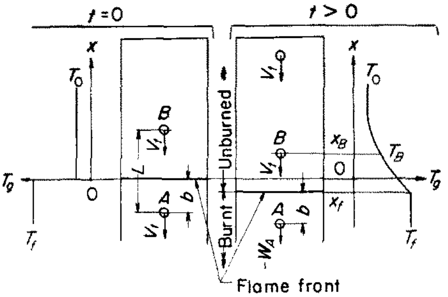


FIG. 8. Model of flame propagation process.

($t = 0$) when A droplets are ignited. As the air temperature at the position of B plane T_B rises due to thermal conduction from the flame front, the temperature of B droplet T_d also rises, and at time $t = t_{eq}$, reaches the equilibrium temperature (wet-bulb temperature) T_{eq}. At this time, around B droplet has been formed an air-vapor film having a concentration high enough to be ignited, and, therefore, t_{eq} may be taken for a physical ignition lag. B droplet furthermore, after the lapse of chemical ignition lag, is ignited at time $t = t_{ig}$. If the value of t_{ig} is known, the velocity of flame propagation V_f will be given by $V_f = L/t_{ig} - V_t$. Besides, if it is assumed that, in order for B droplet to be ignited to have a stable combustion, it should be ignited considerably before A droplet is burnt out, then, the value of L which gives $t_{ig} = \theta\tau$ (θ is a positive constant smaller than 1) will coincide with the interdroplet distance of the mixture at the lower limit of flammability, thus the lower limit may be predicted. Where the velocity of flame propagation V_f must be a positive finite value.

Thus, a model of flame propagation, which, though simplified, is concrete, has been composed.

Governing equations

Distance between burning droplets and flame front. With relatively large liquid drops, the following relation is found between the diameter of the flame surrounding a droplet D_f and the diameter of the droplet d [4]:

$$D_f/d = \tfrac{1}{2}\,[1 + \sqrt{(1 + 2k^*/d)}] \qquad (1)$$

Where k^* is a function of the thermodynamic properties of the fuel vapor. Therefore, in reference to the above equation, the distance between the burning droplets and flame front b can be calculated as:

$$b = \tfrac{1}{4}\,d_0[1 + \sqrt{(1 + 2k^*/d_0)}] \qquad (2)$$

In equation (2), however, for simplicity, k^* is considered to be a constant. Now, in order to estimate the approximate value of k^*, if the value $D_f = 1{\cdot}02$ mm for $d = 0{\cdot}053$ mm obtained by Burgoyne and Cohen [3] with tetralin droplet suspension, is substituted in equation (2), 37 mm will be obtained as the value of k^*.

Equation of heat conduction. As thermal expansion of gas is ignored here, if it is assumed that the plane-shaped flame appears at the position of the origin at time $t = 0$, the equation of heat conduction is:

$$\frac{\partial T_g}{\partial t} = a\,\frac{\partial^2 T_g}{\partial x^2} \qquad (3)$$

In this equation, as the air is assumed not to expand, specific heat at constant volume must be used in thermal diffusivity, i.e. $a = \lambda/\gamma_g c_v$. For simplicity, a is taken as a constant value. The initial condition and boundary condition are:

$$\left.\begin{aligned} t \leqq 0:&\quad T_g = T_0\,(x > 0)\\ x = x_f:&\quad T_g = T_f\,(t \geqq 0)\end{aligned}\right\} \qquad (4)$$

Now, the position of the flame front x_f will be treated. As the distance between A plane and flame front b is taken as constant and suffers no change with time, the flame front is supposed to move at the same velocity with A droplets. On the other hand, the velocity of A droplets w_A (the direction of x-axis, i.e. upward direction is taken as positive), may be represented by the following equation, as recognized from Fig. 7:

$$w_A = -V_t(1 - t/\tau) \qquad (5)$$

Integrating:

$$-\frac{x_f}{V_t\tau} = \frac{t}{\tau}\left(1 - \frac{t/\tau}{2}\right) \qquad (6)$$

For the convenience of calculation, the right-hand side of the above equation is approximated by $\sqrt{(t/\tau)}/2$, and, introducing ξ_f defined by $\xi_f = x_f/\sqrt{t}$:

$$\xi_f = x_f/\sqrt{t} = -V_t(\sqrt{\tau})/2 = \text{const.} \qquad (7)$$

As the flame temperature T_f has been assumed to be a constant, if the variable ξ, defined in equation (8), is introduced, equation (3) is converted to an ordinary differential equation, such as equation (9):

$$\xi = x/\sqrt{t} \tag{8}$$

$$\frac{d^2T_g}{d\xi^2} + \frac{\xi}{2a}\frac{dT_g}{d\xi} = 0 \tag{9}$$

The boundary conditions are:

$$\left.\begin{array}{ll} \xi = \xi_f: & T_g = T_f \\ \xi \to \infty: & T_g \to T_0 \end{array}\right\} \tag{10}$$

Solving equation (9) under the above conditions:

$$T_g = T_f - \frac{T_f - T_0}{1 - \operatorname{erf}(\xi_f/2\sqrt{a})} \left\{ \operatorname{erf}\left(\frac{\xi}{2\sqrt{a}}\right) - \operatorname{erf}\left(\frac{\xi_f}{2\sqrt{a}}\right) \right\} \tag{11}$$

Where:

$$\operatorname{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y \exp[-y^2]\,dy\text{: error function.}$$

On the other hand, B plane, which was located at $x = L - b$ at $t = 0$, moves downwards at a constant velocity V_t until B droplets are ignited. Therefore, the position of B plane x_B will be:

$$x_B = L - b - V_t t \tag{12}$$

Dividing both sides by $\sqrt{t}$, and introducing ξ_B defined by $\xi_B = x_B/\sqrt{t}$:

$$\xi_B = (L - b)/(\sqrt{t}) - V_t\sqrt{t} \tag{13}$$

Then T_B may be obtained by putting $\xi = \xi_B$ in equation (11). The terminal settling velocity of droplets, if the drag force exerted on droplets is to be calculated approximately by using Stokes's law, may be obtained by the following equation:

$$V_t = \frac{1}{18} d_0^2 \left(\frac{\gamma_l}{\mu}\right)_{T=T_0} \tag{14}$$

Temperature rise of B droplet. When T_B is obtained, ignition lag of B droplets may be calculated. The ignition lag, as mentioned earlier, is divided into two parts, the physical lag until B droplets reach the equilibrium temperature T_{eq} and the chemical lag until they subsequently come to ignition.

With regard to the temperature of droplets T_d, as the evaporation of droplets and the radiative heat transfer from the flame to the droplets are ignored, the following equation is obtained:

$$\frac{\pi}{6} d_0^3 \gamma_l c_l \frac{dT_d}{dt} = \alpha_c \pi d_0^2 (T_B - T_d) \tag{15}$$

Where α_c is the coefficient of heat transfer. Ranz and Marshall [5] experimentally found out the following relation in case there exists a relative velocity between a droplet and air:

$$Nu = 2{\cdot}0 + 0{\cdot}60\,Pr^{1/3}\,Re^{1/2} = 2K \tag{16}$$

Substituting the value of α_c obtained from this equation in equation (15):

$$\frac{dT_d}{dt} = \frac{12K\lambda}{d_0^2 \gamma_l c_l}(T_B - T_d) \tag{17}$$

The initial condition is:

$$t = 0{:}\quad T_d = T_0 \tag{18}$$

Equilibrium temperature of B droplet. In this case, as the air temperature T_B around droplets changes with time, strictly speaking, no equilibrium temperature exists. Accordingly, a hypothetical equilibrium temperature will be considered, on the assumption that steady-state evaporation takes place in accordance with T_B at each instant even if T_B were changed, and that no interference occurs between droplets. Then, since effects of pressure rise of the air consequent upon temperature rise and of radiative heat transfer are ignored, the relation between T_{eq} and T_B may be calculated by the usual method [6].

Chemical ignition lag. Very little is known yet about the chemical ignition lag of a liquid droplet. On referring to Sakai and Kotake's semi-empirical formula [7], "chemical ignition lag" $= Z^* \exp[E/\mathscr{R}T_g]$, obtained for liquid fuel sprays injected into a high-temperature electric furnace whose temperature is T_g, the ignition of B droplet is assumed to take place when the value of Z defined by:

$$Z = \int_{t_{eq}}^{t} \exp[-E/\mathscr{R}T_B]\,dt \tag{19}$$

reaches a definite value Z^*, this time being symbolized by t_{ig}.

Flame propagation velocity and lower limit of flammability. When the ignition time of B droplets t_{ig} is obtained, the flame propagation velocity V_f, as described previously, may be calculated as follows:

$$V_f = L/t_{ig} - V_t \tag{20}$$

The value of L in this equation may be easily obtained from the concentration of the mixture (weight of droplets contained in a unit volume of the mixture).

In regard to the lower limit of flammability, the following two conditions may be considered to define it:

(1) Ignition of B droplets must take place considerably before A droplets are burnt out. Accordingly, $t_{ig} \leqq \theta_T$. Where θ is a positive constant smaller than 1.

(2) The flame propagation velocity V_f must be a positive finite value.

Of the above, (2) is the condition for droplets being constantly ignited in unvitiated air. A mixture of lowest concentration satisfying these two conditions will give the concentration at the lower limit of flammability $C_{d,\,\min}$.

Thus, the governing equations applicable to the case when thermal expansion of gas is ignored, has been obtained. A similar theoretical analysis has been carried out also for the case when thermal expansion of gas under a constant pressure was considered. As no major difference exists in the basic principle, its description will be skipped here. About the case when flame propagates horizontally, description will be provided later.

Scope of application of the theory

The above theoretical analysis concerned the phenomenon of relay transfer of a laminar flame through a droplet suspension of liquid fuel of a pure substance, having a uniform diameter. It is known, however, that such relay transfer in dilute mixtures may be seen only above 30 or 40 μ, as mentioned previously. Accordingly, the lower limit of the range of diameter, to which this theory can be applied, is known to be 30 or 40 μ in case of dilute mixtures.

On the other hand, as the droplet diameter increases, its sedimentation velocity rapidly increases. Since, however, an implicit assumption that the sedimentation velocity is not very much larger than the propagation velocity of flame is included in this theory, it may be assumed that this theory also comprises an upper limit in its scope of application. The value of such upper limit is assumed to be 80 or 100 μ.

Besides, if B droplets are permitted to approach too near to the flame front, thermal conduction may not be treated one-dimensionally. This indicates that this theory may not be applied when the concentration of the mixture is too high.

This theory may be applied, after all, only to the case when a laminar flame propagates discontinuously through a droplet suspension of liquid fuel of a pure substance having a uniform diameter. Besides, it is known to be desirable that the diameter ranges between 30 ∼ 100 μ and the mixture does not have unduly high concentration.

An example of calculation

As an example of application of the above theory, the flame propagation velocity and the concentration at the lower limit of flammability are calculated, pertaining to the case when a mixture of tetralin droplets having a uniform diameter and dry air, 15°C and 1 atm, is filled in a vertical tube, of which the upper end is closed and the bottom is opened, and, the flame propagates upwards one-dimensionally.

Referring to the previous studies, numerical values assumed or determined as follows:

$T_0 = 288°\text{K}$, $p = 1$ atm,
$E/\mathscr{R} = 8000°\text{K}$ [7], $Z^* = 5 \times 10^{-11}$ h,†
$K_b = 0{\cdot}5\ \text{mm}^2/\text{s}$ [3], $k^* = 37$ mm,‡
$\theta = 0{\cdot}7$.

The value of the flame temperature T_f is to be chosen appropriately in accordance with the interdroplet distance L and droplet diameter.

---

† The value of Z^* is here chosen as considerably bigger than in the case of spray. This is for the reason that ignition lag of a single droplet is supposed to be far greater than ignition lag of spray.

‡ Cited previously.

For simplicity, however, it is put here at the constant value of 2800°K.†

Here, the process of calculation will be omitted and mention will be made only of the results of calculation, which will be compared with measured values.

Figure 9 shows the effect of the droplet diameter and the mixture concentration on the velocity of flame propagation. The solid line denotes the calculated value based on the above theory, in which thermal expansion of gas is ignored, while the broken line refers to the calculated value based on the theory assuming thermal expansion of gas under a constant pressure. The chain line represents the lower limit of flammability. It is known that the propagation velocity is smaller for larger droplet diameter, that it rapidly increases as the diameter is reduced and that the propagation velocity of flame increases more or less linearly, along with the mixture concentration for a fixed diameter.

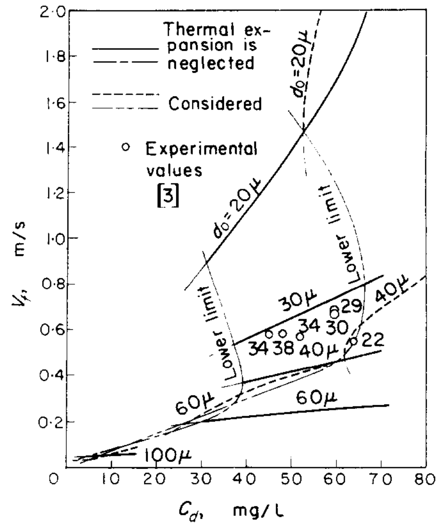


FIG. 9. Flame propagation velocity.

As these two calculated values are compared, it is realized that they coincide fairly well at 100 μ, but, as the diameter becomes smaller, they come to have an outstanding difference, and, from this fact, it is known that, when the diameter is small, thermal expansion has greater effect, while, when the diameter grows larger, this effect is reduced. In this diagram, the measured values obtained by Burgoyne and Cohen [3] are also plotted, except those of the case of diameter being below 20 μ, where homogeneous transfer of flame takes place. Where the numerical figures accompanying the measured points represent the droplet diameter. Comparing the measured values with the theoretical ones, it is found that, in the range of 30 ~ 40 μ, where the nature of relay transfer of flame is predominant, the measured and calculated values coincide fairly well quantitatively. Moreover, it seems that the values calculated ignoring thermal expansion are in better agreement with the measured ones.

Figure 10 shows the relation between the droplet diameter and the concentration at the lower limit of flammability. The calculated values in case thermal expansion of is ignored is represented by a solid line, and those in case thermal expansion under a constant pressure is assumed, by a broken line. In this diagram, the measured values of Fig. 3 are also replotted for the convenience of comparison. Comparing the measured values with the calculated ones, both are found to well

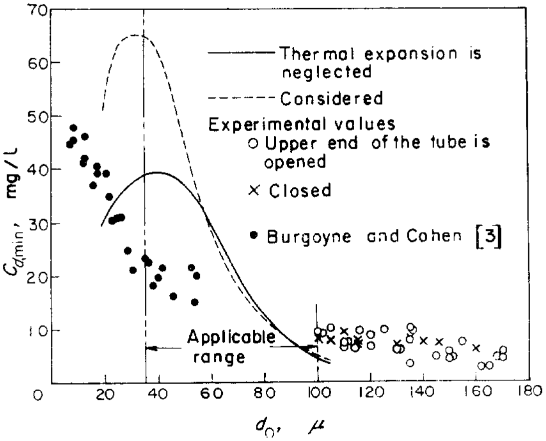


FIG. 10. Lower limit of flammability.

---

† This is a theoretical value obtained by neglecting reaction-rate and dissociation, so, naturally, is considerably higher than the actual flame temperature. However, it is assumed to be more suitable for the purpose of calculating the temperature distribution at a considerable distance from the flame front, in ignoring recombination and diffusion. Besides, calculation by T_f = 2000°K yielded no major difference in the result obtained.

coincide qualitatively in the applicable range of this theory, the concentration at the lower limit rapidly decreasing as the diameter increases. While, quantitatively, a considerable difference is noted, this may be attributed to the fact that the theoretical analysis was only approximate, and, besides, that droplets, actually dispersed at random, were approximated to the arrangement of simple cubic lattice and, moreover, that the flame front, actually, is not horizontal, but is inclined. Now, when two kinds of calculated values are compared, it is found that both coincide well when the diameter is large but that they show a increased difference when it becomes smaller. From this fact, again, it may be concluded that thermal expansion has less effect as the diameter is larger.

EFFECT OF PROPAGATION DIRECTION OF FLAME

Here, the effect of the sedimentation of droplets will be clarified by comparing upward propagation of flame and its horizontal propagation.

The governing equations for the case of horizontal propagation may be easily introduced by substituting $g = 0$, $V_t = 0$ in those for upward propagation. Figures 11 and 12 show the results of calculation carried out for horizontal propagation by using the same numerical values as were used in the previous example. In this case, the theory considering thermal expansion of gas under a constant pressure is employed and the mixture is assumed to be contacting a solid wall at a great distance ahead from the flame.

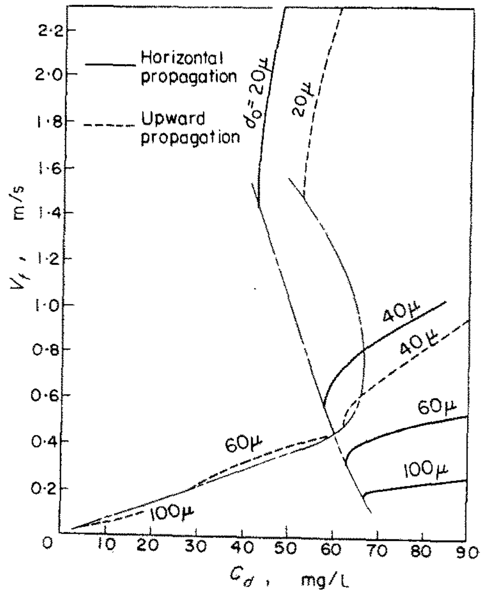


FIG. 11. Effect of direction of flame propagation on propagation velocity.

Figure 11 illustrates the calculated values of the propagation velocity of flame. In the diagram, the solid line refers to a horizontal propagation of flame and the broken line to an upward propagation and the chain line to the lower limit of flammability. From the diagram, it is known that the greater the diameter is, the greater will be the effect of propagation direction. Even in case of large droplet suspensions, however, no great difference in the propagation velocity is seen in so far as the concentration is the same, the only difference being in the shape of the curve and the concentration at the lower limit.

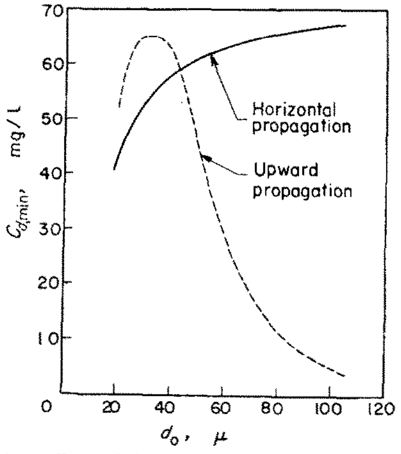


FIG. 12. Effect of direction of flame propagation on the lower limit of flammability.

Figure 12 relates to a comparison of the calculated values of the concentration at the lower limit, in which the solid line represents horizontal propagation and the broken line upward propagation. In the diagram, when the diameter is small, only a slight difference is seen between them, but, as the diameter increases, the concentration at the lower limit rapidly decreases in case of upward propagation, while, in case of horizontal propagation, it tends to increase slightly in proportion as the diameter becomes larger.

Thus, from the fact that the effect of direction of flame propagation becomes larger when the droplet diameter, and so its velocity of sedimentation grow larger, it is known that the influence of the sedimentation is considerably large.

EFFECT OF CONCENTRATION OF OXYGEN IN AIR

When the mixture of tetralin droplets having a uniform diameter and the air, is diluted by nitrogen, its concentration at the lower limit will naturally rise, but, between 10 ~ 45 μ, the effect of dilution will become less as the diameter grows larger, and around 45 μ, the droplet suspensions, even in case the concentration of oxygen is extremely weak, are known to be flammable [3]. No detailed report is on hand on the range of larger diameters, but it has been found that there exists a diameter where the effect of oxygen concentration is minimum, and that, if the diameter is increased further, the effect again gets larger, and that, at 200 μ, flammability will be lost at an oxygen concentration considerably higher than in case of a homogeneous mixture of hydrocarbon vapor [8].

Here, an attempt will be made to explain this fact, theoretically.

Effects of oxygen concentration on various factors

Of a single droplet burning in a large space, many investigators have so far carried out theoretical calculations of the burning constant, and the temperature and diameter of the flame surrounding the droplet. While these results may not be adopted quantitatively in this case, it is assumed here that the qualitative tendency only may be applied. Here, approximate theoretical formulae of Wise *et al.* [9] will be employed, and, in consideration of $C_O/z \ll 1$, $(T_d - T_g) \ll (H - r)/c_p$, the following equations will be obtained for K_b, T_f and b of droplet suspensions:

$$K_b = k_1 \ln [(H/rz)C_O] \qquad (21)$$

$$T_f = k_2 C_O + T_d \qquad (22)$$

$$b = k_3 \ln [(H/rz)\, C_O]/\ln [1 + (1/z)\, C_O] \qquad (23)$$

Where k_1, k_2, k_3 are proportional constants.

For tetralin droplet suspensions, $H = 10\,500$ kcal/kg, $r = 82{\cdot}4$ kcal/kg and $z = 3{\cdot}15$ kg/kg were taken. The values of k_1, k_2, k_3 were chosen so that the values of K_b, T_f and b in the standard atmosphere ($C_O = 0{\cdot}2314$ kg/kg) may equal the values used in the previous examples. Then, the variations of K_b, T_f and b with oxygen concentration were calculated, the result of which is shown in Fig. 13.

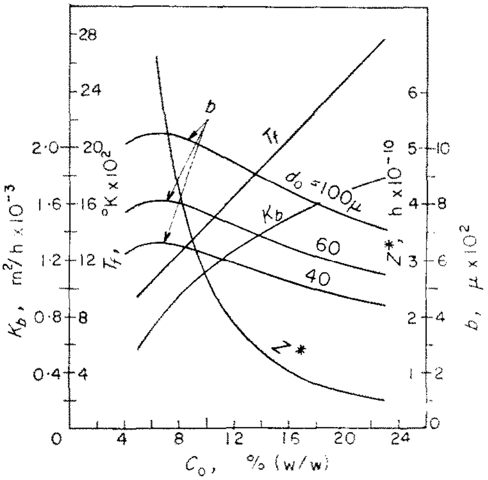


FIG. 13. Variations of various factors with oxygen concentration.

Next, consideration will be given to the chemical ignition lag of droplets. Mullins [10] represented the ignition lag of spray t_{ig} by the following empirical formula:

$$t_{ig} = A^* \exp (b^*/T_g)$$

Where A^*, b^* are constants, T_g is the ambient temperature. Mullins found out that, as the concentration of oxygen C_O decreases, constant A^* increases in counterproportion to the square of C_O. With this in consideration, it is assumed here that, between Z^* and C_O, the following relation exists:

$$Z^* = k_4 C_O^{-2} \qquad (24)$$

Where k_4 is a constant. The activation energy E in equation (19) is assumed to be independent of the concentration of oxygen. Here, again, the value of k_4 was so determined that the value of Z^* in the standard atmosphere may equal the value used in the previous examples, and, the result of calculation thus carried out about the relation between Z^* and C_O are given in Fig. 13.

The changes of the thermal properties of the air in case the concentration of oxygen is reduced is so small that they are ignored here.

Example of calculation

The case when the mixture of tetralin droplets of a uniform diameter and the air is diluted by nitrogen is here considered. It is assumed that flame propagates upwards one-dimensionally, and calculation is carried out by using the theory ignoring thermal expansion. For those properties, which change depending on the concentration of oxygen, are used the values shown in Fig. 13, and for the others the same values as in the previous examples. As the concentration of oxygen decreases, the propagation velocity of flame also decreases. Now, the critical velocity above which stable propagation takes place is set at 4 cm/s, it being assumed that no flame propagation takes place at a velocity below it.

The process of calculation being omitted, the propagation velocity obtained for 40, 60 and 100 μ, respectively, with the concentration of oxygen changed, is shown in Fig. 14. By the broken line in the diagram the lower limit of flammability is represented. This diagram shows that, as the concentration of oxygen reduces, the propagation velocity of flame rapidly decreases. After, however, the curves of the lower limit cross the straight line of the critical velocity, the lower limit comes to be defined by the critical velocity, and, thus, the curves of the lower limit bend here abruptly to coincide with this straight line. The concentration at the lower limit is replotted in Fig. 15.

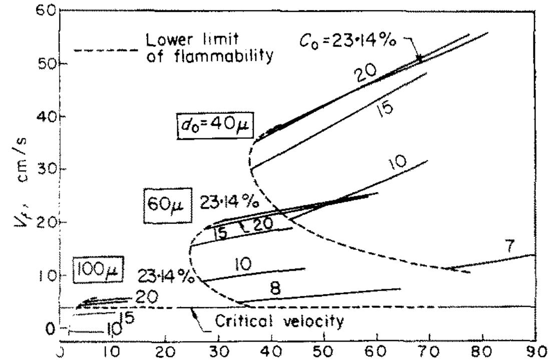


FIG. 14. Effect of oxygen concentration on flame propagation velocity.

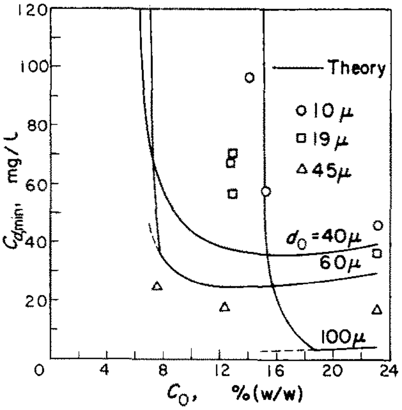


FIG. 15. Effect of oxygen concentration on the lower limit of flammability.

In this diagram, each of the curves for diameters of 60 and 100 μ, which otherwise may show a gradual change, is seen to bend at a certain point and rise upwards abruptly. This signifies that the conditions defining the lower limit are exchanged here. The measured values obtained by Burgoyne and Cohen [3] are also given in this diagram. Of these values, those for 10 and 19 μ do not apply to relay transfer treated in this study, and, therefore, can not be directly compared with the calculated values. From the measured for 45 μ, however, it is found that the concentration at the lower limit is kept approximately constant until the oxygen concentration falls to 10 per cent, and then indicates a slight increase at the value of 8 per cent. In this respect, they qualitatively coincide fairly well with the calculated values for 40 and 60 μ. As the droplet diameter gets larger, it has been found experimentally that the lower limit again becomes much susceptible to the oxygen concentration [8]. The calculated values show the same tendency, that is, no major difference in qualitative tendency is seen between the calculated values for 40 and 60 μ. But, as the diameter gets larger up to 100 μ, the lower limit becomes greatly sensitive to the concentration of oxygen, flammability being lost at approximately the same concentration of oxygen as measured with the droplet suspensions of 10 μ.

REFERENCES

1. E. K. Čekalin, Propagation of flame in turbulent-flow of two-phase fuel-air mixture, in *Eighth Symposium (International) on Combustion*, pp. 1125–1129. Williams & Wilkins, Baltimore (1962).
2. K. R. May, The measurement of airborne droplets by the magnesium oxide method, *J. Sci. Instrum.* **25**, No. 5, 128–130 (1950).
3. J. H. Burgoyne and L. Cohen, The effect of drop size on flame propagation in liquid aerosols, *Proc. Roy. Soc.* **A225**, 357–392 (1954).
4. C. C. Miesse, On the combustion of a liquid fuel spray, in *Sixth Symposium (International) on Combustion*, pp. 732–738. Reinhold, New York (1957).
5. W. E. Ranz and W. R. Marshall Jr., Evaporation from drops, *Chem. Engng Progr.* **48**, 141–152, 173–180 (1952).
6. J. P. Longwell, Combustion of liquid fuels, *Combustion Processes* (B. Lewis, R. N. Pease and H. S. Taylor, editors), pp. 407–443. Princeton University Press (1956).
7. Tadami Sakai and Susumu Kotake, Ignition lag of fuel spray [1], *Sci. Mach.* **10**, 383–387 (1958).
8. J. H. Burgoyne, Mist and spray explosions, *Chem. Engng Progr.* **53**, 121–124 (1957).
9. H. Wise, J. Lorell and B. J. Wood, The effect of chemical and physical parameters on the burning rate of a liquid droplet, *Fifth Symposium (International) on Combustion*, pp. 132–141. Reinhold, New York (1955).
10. B. P. Mullins, The spontaneous ignition of fuels injected into a hot air stream. I. Development of a combustion test rig for measuring the ignition delay of fuels, *Fuel, Lond.* **32**, 211–233 (1953).

Résumé—Comme première étape dans la clarification du mécanisme de propagation de flamme dans un jet en brouillard de combustible liquide, une étude a été faite dans le cas d'une flamme laminaire se propageant à travers les suspensions de gouttelettes d'un combustible liquide, ayant des diamètres uniformes. Initialement, afin d'examiner la nature de la propagation de la flamme, des suspensions de gouttelettes de tétraline ayant des diamètres approximativement uniformes ont eté préparées, et leur concentration à la limite la plus faible d'inflammabilté a été examiné, et, en même temps, l'apparence de la flamme et le processus de sa propagation ont été observés. Sur la base de cette observation, un modèle de propagation de flamme a été proposé, selon lequel les gouttelettes brûlant au front de flamme chaffent le mélange non brûlé, les gouttelettes non brûlées étant alors allumées après un certain retard à l'allumage. En utilisant ce modèle, une analyse théorique a été conduite, et la vitesse de propagation de flamme et la concentration à la limite inférieure ont été calculées. Les valeurs prédites montraient un accord satisfaisant avec les résultats expérimentaux, au moins qualitativement. A partir de cette analyse, on a trouvé que la sédimentation des gouttelettes exerce une grande influence sur le processus de propagation de flamme, et que, dans le cas où le mélange contenant les gouttelettes est dilué par de l'azote, son effet sur la limite inférieure dépend grandement du diamètre des gouttelettes, comme ce qui avait déjà été trouvé expérimentalement.

Zusammenfassung—Als erster Schritt zur Klärung des Ausbreitungsvorgangs einer Flamme in einem zerstäubten Strahl eines flüssigen Brennstoffes wurde eine Untersuchung für den Fall der laminaren Flammenausbreitung in Suspensionen mit Tröpfchen eines flüssigen Brennstoffes, die gleichen Durchmesser haben, ausgeführt. Ursprünglich wurden, um die Natur der Flammenausbreitung zu untersuchen, Suspensionen von Tetralintröpfchen annähernd gleichen Durchmessers vorbereitet und ihre Konzentration an der unteren Zündgrenze untersucht; gleichzeitig wurde dabei das Erscheinen der Flamme und ihr Ausbreitungsprozess beobachtet. Mit dieser Beobachtung als Grundlage wurde ein Modell der Flammenausbreitung unter Berücksichtigung des Phänomens vorgeschlagen, dass die brennenden Tröpfchen an der Flammenfront das unverbrannte Gemisch erhitzten und dass dann die unverbrannten Tröpfchen nach Ablauf einer bestimmten Zündverzögerung entflammen. Unter Verwundung dieses Modells wurde eine theoretische Analyse erstellt und die Fortpflanzungsgeschwindigkeit der Flamme und die Konzentration an der unteren Grenze berechnet. Die vorhergesagten Werte zeigten zufriedenstellende Übereinstimmung mit den Versuchsergebnissen, zumindest qualitativ. Aus dieser Analyse ergab sich, dass ein Absetzen von Tröpfchen grossen Einfluss auf den Vorgang der Flammenausbreitung ausübt und dass im Falle einer Verdünnung des Gemisches, welches die Tröpfchen enthält, mit Stickstoff die Wirkung auf die untere Grenze stark vom Tröpfchendurchmesser abhängt, wie schon experimentell nachgewiesen war.

Аннотация—С целью выяснения механизма распространения пламени при распылении жидкого топлива проводилось исследование распространения ламинарного пламени во взвеси капель жидкого топлива одинакового диаметра. Для исследования природы распространения пламени сначала приготовлялась суспензия тетралина с каплями одинакового диаметра. Рассматривалась их концентрация в нижнем пределе воспламеняемости, и одновременно наблюдалось возникновение пламени и процесс его распространения. На основе этих наблюдений предложена модель, учитывающая, что капли горящие на фронте пламени, нагревают негорящую смесь, а затем, спустя некоторое время, негорящие капли воспламеняются. С помощью этой модели проведен теоретический анализ, а также вычислены скорость распространения пламени и концентрация в нижнем пределе. Расчетные величины показали хорошее согласование с экспериментом, по крайней мере, качественное. В результате этого анализа было выяснено, что осаждение капель оказывает бошое влияние на процесс распространения пламени, а также то, что в случае, если смесь, содержащая капельки, разбавлена азотом, его влияние на нижний предел существенно зависит от диаметра капель, что уже было установлено экспериментально.